

## STUDY MATERIAL AND PRACTICE ASSIGNMENT IV MOVING CHARGES AND MAGNETISM

### INTRODUCTION

In 1820 Hans Christian Oersted noticed that a current in a straight wire caused a noticeable deflection in a nearby magnetic compass needle. He also found that the alignment of the needle is tangential to an imaginary circle which has the straight wire as its centre and has its plane perpendicular to the wire. This situation is depicted in

the [Fig.(a)]

Iron filings sprinkled around the wire arrange themselves in concentric circles with the wire as the centre [Fig.(b)].

Oersted concluded that **moving charges or currents produced a magnetic field in the surrounding space.**

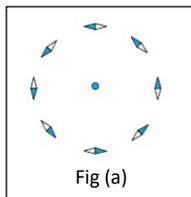


Fig (a)

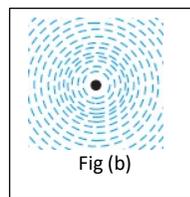


Fig (b)

### THE MAGNETIC FIELD

Magnetic field is the region surrounding a magnet or moving charge or electric current in which the magnetic effects are perceptible. Magnetic field intensity is a vector quantity and also known as magnetic induction vector. It is

represented by  $\vec{B}$ . And it is also called magnetic flux density or magnetic field induction. The SI unit of magnetic

field is weber/m<sup>2</sup> or tesla (T) in the cgs system its unit is gauss. **1 weber/m<sup>2</sup> = 1T = 10<sup>4</sup> gauss**

The earth's magnetic field is about  $3.6 \times 10^{-5}$  T

Dimensional Formula of **B** is given by **[B]=[F/qv]=[M<sup>1</sup>T<sup>-2</sup>A<sup>-1</sup>]**

### Lorentz Force

The electromagnetic force acting on a moving charge particle in electromagnetic field is known as Lorentz

force which is given by  $\vec{F} = q[\vec{E} + \vec{v} \times \vec{B}]$

### Magnetic Lorentz Force

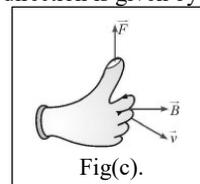
Is the magnetic force acting on a charge particle moving in the magnetic field which is given by  $\vec{F} = q[\vec{v} \times \vec{B}]$

(i) It depends on  $q$ ,  $v$  and  $\mathbf{B}$  (charge of the particle, the velocity and the magnetic field). Force on a negative charge is opposite to that on a positive charge.

(ii) The magnetic force  $q[\vec{v} \times \mathbf{B}]$  vanish (become zero) if velocity and magnetic field are parallel or anti-parallel.

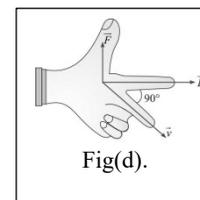
(iii) The force acts in direction perpendicular to both the velocity and the magnetic field. Its direction is given by the screw rule or right hand rule for vector (or cross) product or by Fleming's left hand rule.

**1. Right hand rule :** If we wrap the fingers of the right hand around the line perpendicular to the plane of  $\mathbf{v}$  and  $\mathbf{B}$  so that they curl around from  $\mathbf{v}$  to  $\mathbf{B}$  through smaller angle then thumb points in the direction of force  $\mathbf{F}$ , as illustrated in Fig(c).



Fig(c).

**2. Fleming's left hand rule (FLHR) :** For positive charge if we hold fore finger, middle finger and thumb of the left hand mutually perpendicular to each other; such that fore finger shows the direction of  $\mathbf{B}$ , the middle finger shows the direction of the component of velocity perpendicular to  $\mathbf{B}$ , then thumb indicates the direction of force, as illustrated in Fig(d).



Fig(d).

(iii) The magnetic force is zero if charge is not moving (as then  $|v|=0$ ).

(iv) The magnetic force will be maximum, for  $\theta=90^\circ$ , which is  $F = qvB \sin 90^\circ = qvB$

### NOTE :

- $\vec{F} \perp \vec{v}$  and also  $\vec{F} \perp \vec{B}$
- $\therefore \vec{F} \perp \vec{v} \therefore$  power due to magnetic force on a charged particle is zero. (use the formula of power  $P = \vec{F} \cdot \vec{v}$  for its proof)
- Since the  $\vec{F} \perp \vec{B}$  so work done by magnetic force is zero in every part of the motion. The magnetic force cannot increase or decrease the speed (or kinetic energy) of a charged particle. Its can only change the direction of velocity.

□

**Ex. 1** An electron is moving with a velocity  $(2\hat{i} + 3\hat{j})$  m/s in an electric field of intensity  $(3\hat{i} + 6\hat{j} + 2\hat{k})$  V/m and a magnetic field of  $(2\hat{j} + 3\hat{k})$  tesla. Find the magnitude and direction of the Lorentz force acting on the electron.

**Sol.** Lorentz force is given by

$$\begin{aligned}\vec{F} &= q(\vec{E} + \vec{v} \times \vec{B}) \\ &= -1.6 \times 10^{-19} \left[ (3\hat{i} + 6\hat{j} + 2\hat{k}) + (2\hat{i} + 3\hat{j}) \times (2\hat{j} + 3\hat{k}) \right] \\ &= -1.6 \times 10^{-19} \left[ 3\hat{i} + 6\hat{j} + 2\hat{k} + 9\hat{i} - 6\hat{j} + 4\hat{k} \right] \\ &= -1.6 \times 10^{-19} \left[ 12\hat{i} + 6\hat{k} \right] = 9.6 \times 10^{-19} (2\hat{i} + \hat{k})\end{aligned}$$

$$\therefore F = 9.6 \times 10^{-19} \sqrt{2^2 + 1^2} = 2.15 \times 10^{-18} \text{ N}$$

It is in  $xz$ -plane, making an angle  $\theta$  with the  $x$ -axis, where

$$\theta = \cos^{-1} \frac{2}{\sqrt{5}}$$

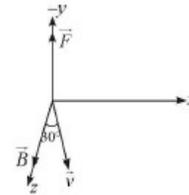
**Ex. 2** A proton beam moves through a region of space where there is a uniform magnetic field of magnitude 2.0 T, with direction along the positive  $z$ -axis as shown in fig.5.7. The protons have velocity of magnitude  $3 \times 10^5$  m/s in the  $xz$  plane, at an angle  $30^\circ$  to

the positive  $z$ -axis. Find force on proton. ( $q = 1.6 \times 10^{-19}$  C)

**Sol.**

The magnitude of force

$$\begin{aligned}F &= qv_{\perp}B \\ &= q \times v \sin 30^\circ \times B \\ &= (1.6 \times 10^{-19}) \times (3 \times 10^5) \times \left(\frac{1}{2}\right) \times (2) \quad \text{Fig. 5.7} \\ &= 4.8 \times 10^{-14} \text{ N}\end{aligned}$$



**Ex. 3** An experimenter's diary reads as follows : "a charged particle is projected in a magnetic field of  $(7.0\hat{i} - 3.0\hat{j}) \times 10^{-3}$  T. The acceleration of a particle is found to be  $(\square\hat{i} + 7.0\hat{j}) \times 10^{-6}$  m/s<sup>2</sup>". The number to be left of  $\hat{i}$  in the last expression was not readable. What can this number be ?

**Sol.**

As magnetic force is perpendicular to the magnetic field, so

$$\vec{B} \cdot \vec{F} = 0$$

$$\text{or } \vec{B} \cdot \vec{a} = 0.$$

Let value in box provided is  $k$ .

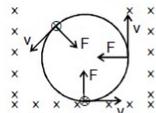
$$\therefore 7k - 3 \times 7 = 0 \Rightarrow k = 3 \quad \text{Ans.}$$

#### MOTION OF A CHARGED PARTICLE IN A UNIFORM MAGNETIC FIELD.

##### WHEN THE CHARGED PARTICLE IS GIVEN VELOCITY PERPENDICULAR TO THE FIELD

Let a particle of charged  $q$  and mass  $m$  is moving with a velocity  $v$  and enters at right angles to a uniform magnetic field  $\vec{B}$  as shown in figure.

The force on the particle is  $qvB$  and this force will always act in a direction perpendicular to  $v$ . Hence, the particle will move on a circular path. If the radius of the path is  $r$  then



$$\frac{mv^2}{r} = Bqv \quad \text{or, } r = \frac{mv}{qB} \quad \dots(10)$$

Thus, radius of the path is proportional to the momentum  $mv$  of the particle and inversely proportional to the magnitude of magnetic field.

**Time period** : The time period is the time taken by the charged particle to complete one rotation of the circular path which is given by,

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB} \quad \dots(11)$$

The time period is independent of the speed  $v$ .

**Frequency** : The frequency is number of revolution of charged particle in one second, which is given by,

$$v = \frac{1}{T} = \frac{qB}{2\pi m} \quad \dots(12)$$

and angular frequency  $= \omega = 2\pi v$

**Ex 4.** A proton ( $p$ ),  $\alpha$  particle and deuteron ( $D$ ) are moving in circular paths with same kinetic energies in the same magnetic field. Find the ratio of their radii and time periods. (Neglect interaction between particles).

**Soln.**

$$\begin{aligned}R &= \frac{\sqrt{2mK}}{qB} \\ \therefore R_p : R_\alpha : R_D &= \frac{\sqrt{2mK}}{qB} : \frac{\sqrt{2.4mK}}{qB} : \frac{\sqrt{2.2mK}}{qB} \\ &= 1 : 1 : \sqrt{2}\end{aligned}$$

$$T = 2\pi m / qB$$

$$\begin{aligned}\therefore T_p : T_\alpha : T_D &= \frac{2\pi m}{qB} : \frac{2\pi 4m}{2qB} : \frac{2\pi 2m}{qB} \\ &= 1 : 2 : 2 \quad \text{Ans.}\end{aligned}$$

#### WHEN THE CHARGED PARTICLE IS MOVING AT AN ANGLE TO THE FIELD

In this case the charged particle having charge  $q$  and mass  $m$  is moving with velocity  $v$  and it enter the magnetic field  $B$  at angle  $\theta$  as shown in figure (e). Velocity can be resolved in two components, one along magnetic field and the other perpendicular to it. Let these components are  $v_{\parallel}$  and  $v_{\perp}$

$v_{\parallel} = v \cos \theta$  and  $v_{\perp} = v \sin \theta$ . See figure (f). The parallel component  $v_{\parallel}$  of velocity remains unchanged as it is parallel to  $B$ . Due to the  $v_{\perp}$  the particle will move on a circular path. So the resultant path will be combination of straight-line motion and circular motion, which will be helical as shown in figure(g).

The radius of path is  $(r) = \frac{mv_{\perp}}{qB} = \frac{mv \sin \theta}{qB} \dots(13)$

Time period  $(T) = \frac{2\pi r}{v_{\perp}} = \frac{2\pi m v \sin \theta}{v \sin \theta qB} = \frac{2\pi m}{qB} \dots(14)$

Frequency  $(f) = \frac{Bq}{2\pi m} \dots(15)$

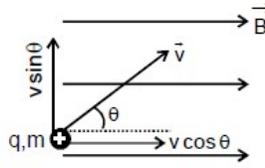
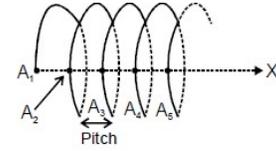


figure (f)



figure(g)

**Pitch :** Pitch of helix described by charged particle is defined as the magnitude of its displacement along the magnetic field in the time in which particle completes one revolution. Pitch = distance A1A2 = A3A4 = ..... =  $v \cos \theta \cdot T$

$$v_{\parallel} \cdot T = v \cos \theta \frac{2\pi m}{Bq} = \frac{2\pi m v \cos \theta}{qB}$$

<p>Ex.5. A beam of protons with a velocity <math>4 \times 10^5</math> m/s enters a uniform magnetic field of 0.3 T at an angle <math>60^\circ</math> to the magnetic field. Find the radius of the helical path taken by the proton beam. Also find the pitch of the helix .  <math>m_p = 1.67 \times 10^{-27}</math> kg</p> <p>Soln. Radius of the helical path taken by the proton beam is given by</p>	$r = \frac{m(v \sin \theta)}{qB} = 1.2 \text{ cm}$ $\text{Time period } T = \frac{2\pi r}{v \sin \theta} = 2.175 \times 10^{-7} \text{ s}$ $\therefore \text{pitch of the helix } p = v \cos \theta \cdot T$ $\Rightarrow p = 4 \times 10^5 \times \frac{1}{2} \times 2.175 \times 10^{-7} = 4.35 \text{ cm}$
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**CYCLOTRON**

Cyclotron is a device which is used to accelerate positive particles like  $\alpha$ -particle, Deuteron etc. It is based on the fact that the electric field accelerates a charged particle and the magnetic field keeps it revolving in circular orbits of increasing radius.

It consists of two hollow D-shaped metallic chambers D1 and D2 called dees. The dees are connected to the source of high frequency electric field. The whole apparatus is placed between the two poles of a strong electromagnet N-S as shown in Fig.(h). The magnetic field acts perpendicular to the plane of the dees.

Every time the particle moves from one dee to another it is acted upon by the electric field. The sign of the electric field is changed alternately in tune with the circular motion of the particle. This ensures that the particle is always accelerated by the electric field. Each time the acceleration increases the energy of the particle. As energy increases, the radius of the circular path increases. So the path is a spiral one.

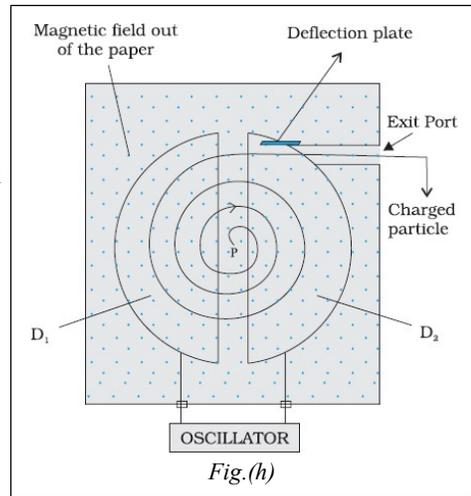
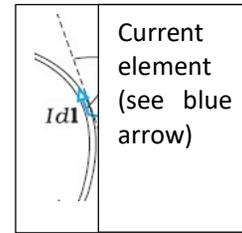


Fig.(h)

<p><b>Cyclotron frequency :</b> It is number of revolutions of charged particle in one second.          Time taken by charged particle to describe a semicircular path is given by</p> $t = \frac{\pi r}{v} = \frac{\pi m}{qB}$ <p>The period of oscillating electric field</p> $T = 2t = \frac{2\pi m}{qB}$ <p>and cyclotron frequency <math>f = \frac{1}{T} = \frac{Bq}{2\pi m}</math>.</p>	<p><b>Maximum kinetic energy of the particle</b>          If <math>r_0</math> = maximum radius = radius of Dee          And <math>v_0</math> = maximum acquired velocity then</p> $r_0 = \frac{mv_0}{qB} \Rightarrow v_0 = \frac{r_0 qB}{m}$ $KE = \frac{1}{2} m v_0^2 = \frac{1}{2} m \left( \frac{r_0 qB}{m} \right)^2 = \left( \frac{q^2 B^2}{2m} \right) r_0^2$ <p>Per second acquired kinetic energy is given by</p> $KE = f \times [qV \times 2] = 2fqV.$
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**Current Element**

A very small element of length  $dl$  of a thin conductor carrying current  $I$  is called current element vector whose magnitude is equal to the product of current and length of small element having the direction of the flow of current.

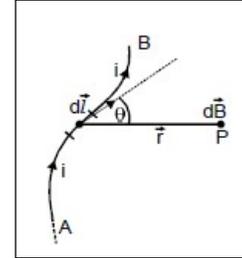


**BIOT-SAVART LAW**

Biot-Savart law gives the magnetic induction due to an infinitesimal current element. Let AB be a conductor of an arbitrary shape carrying a current  $i$ , and P be a point in Vacuum at which the field is to be determined. Let us divide the conductor into infinitesimal current elements.

Let  $\mathbf{r}$  be a position vector from the element to the point P.

According to .Biot-Savart Law., the magnetic field induction  $d\mathbf{B}$  at P due to the current element  $id\mathbf{l}$  is given by



$$d\mathbf{B} \propto I, \quad d\mathbf{B} \propto d\ell, \quad d\mathbf{B} \propto \sin\theta \text{ and } d\mathbf{B} \propto \frac{1}{r^2} \Rightarrow d\mathbf{B} \propto \frac{Id\ell \sin\theta}{r^2} \Rightarrow d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{Id\ell \sin\theta}{r^2}$$

Vector form of Biot-Savart's law

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{Id\vec{\ell} \sin\theta}{r^2} \hat{n} \quad \hat{n} = \text{unit vector perpendicular to the plane of } (Id\vec{\ell}) \text{ and } (\vec{r}) \quad \text{OR} \quad d\vec{B} = \frac{\mu_0}{4\pi} \frac{Id\vec{\ell} \times \vec{r}}{r^3}$$

where

$$\frac{\mu_0}{4\pi} = 10^{-7} \text{ Tm/A}$$

We call  $\mu_0$  the permeability of free space (or vacuum).

&  $\theta$  is the angle between  $d\vec{\ell}$  and  $\vec{r}$

If the medium is other than air or vacuum, the magnetic inducton is

$$d\vec{B} = \frac{\mu_0 \mu_r}{4\pi} \frac{i(d\vec{\ell} \times \vec{r})}{r^3} \quad \dots(2)$$

where  $\mu_r$  is relative permeability of the medium and is a dimensionless quantity.

The sense of  $d\mathbf{l} \times \mathbf{r}$  is also given by the *Right Hand Screw rule* :If we look at the plane containing vectors  $d\mathbf{l}$  and  $\mathbf{r}$  and imagine moving from the first vector towards second vector. If the movement is anticlockwise, the resultant is towards us. If it is clockwise, the resultant is away from us.

**Biot-Savart law similarities and differences with the Coulomb's law**

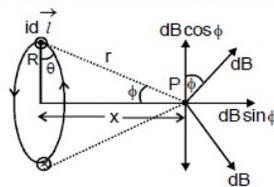
<p>(i) Both are long range, and depend inversely on the square of distance from the source to the point of interest. The principle of superposition applies to both fields.</p> <p>(ii) The electrostatic field is produced by a scalar source, namely, the electric charge. The magnetic field is produced by a vector source <math>I d\mathbf{l}</math> current element vector.</p>	<p>(iii) The electric field is along the position vector joining the source and the field point. The magnetic field is perpendicular to the plane containing the position vector <math>\mathbf{r}</math> and the current element <math>I d\mathbf{l}</math>.</p> <p>(iv) There is an angle dependence in the Biot-Savart law which is not present in the electrostatic case.</p>
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NOTE:1. The magnetic field at any point due to any current element  $i d\mathbf{l}$  in its line is zero. Since  $\sin\theta = 0$  for  $\theta = 0$  or  $\theta = \pi$   
 2. There is an interesting relation between  $\epsilon_0$ , the permittivity of free space;  $\mu_0$ , the permeability of free space; and  $c$ , the speed of light in vacuum.

$$\epsilon_0 \mu_0 = (4\pi\epsilon_0) \left( \frac{\mu_0}{4\pi} \right) = \left( \frac{1}{9 \times 10^9} \right) (10^{-7}) = \frac{1}{(3 \times 10^8)^2} = \frac{1}{c^2}$$

**MAGNETIC FIELD AT AN AXIAL POINT OF A CIRCULAR COIL**

Consider a circular loop of radius  $R$  and carrying a steady current  $i$ . We have to find out magnetic field at the axial point P, which is at distance  $x$  from the centre of the loop.



Consider an element  $i d\vec{l}$  of the loop as shown in figure, and the distance of point P from current element is  $r$ . The magnetic field at P due to this current element from the equation (1) can be given by,

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i d\vec{l} \times \vec{r}}{r^3}$$

In case of point on the axis of a circular coil, as for every current element there is a symmetrically situated opposite element, the component of the field perpendicular to the axis cancel each other while along the axis add up.

$$\therefore B = \int dB \sin \phi = \frac{\mu_0}{4\pi} \int \frac{idl \sin \theta}{r^2} \sin \phi$$

Here,  $\theta$  is angle between the current element  $i d\vec{l}$  and  $\vec{r}$ , which is  $\frac{\pi}{2}$  everywhere and

$$\sin \phi = \frac{R}{r} = \frac{R}{\sqrt{R^2 + x^2}}$$

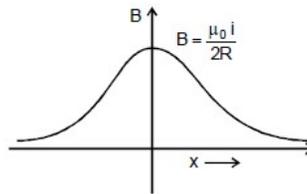
$$\therefore B = \frac{\mu_0}{4\pi} \frac{iR}{(R^2 + x^2)^{3/2}} \int_0^{2\pi R} dL$$

$$\text{or, } B = \frac{\mu_0}{4\pi} \frac{iR}{(R^2 + x^2)^{3/2}} (2\pi R)$$

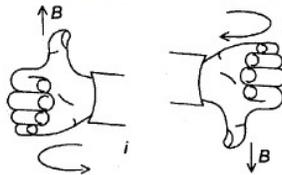
$$\text{or, } B = \frac{\mu_0}{4\pi} \frac{2\pi i R^2}{(R^2 + x^2)^{3/2}} \quad \dots(4)$$

If the coil has N turns, then

$$B = \frac{\mu_0}{4\pi} \frac{2\pi N i R^2}{(R^2 + x^2)^{3/2}}$$



**Direction of  $\vec{B}$**  : Direction of magnetic field at a point the axis of a circular coil is along the axis and its orientation can be obtained by using the right-hand thumb rule. If the fingers are curled along the current, the stretched thumb will point towards the magnetic field.



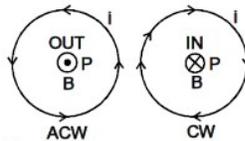
Magnetic field will be out of the page for anticlockwise current while into the page for clockwise current as shown in the figure given.

Now consider some special cases involving the application of equation (4)

**CASE I : Field at the centre of the coil**

In this case distance of the point P from the centre ( $x$ ) = 0, the magnetic field

$$B = \frac{\mu_0}{4\pi} \frac{2\pi i}{R} = \frac{\mu_0}{2} \frac{i}{R}$$



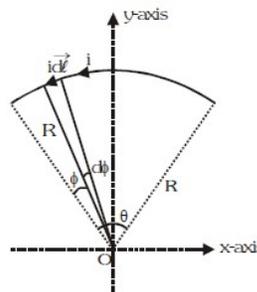
**CASE II : Field at a point far away from the centre**

It means  $x \gg R$ ,  $B = \frac{\mu_0}{4\pi} \frac{2\pi R^2 i}{x^3}$

**Example 6.** Find the magnetic field at the centre of a current carrying conductor bent in the form of an arc subtending angle  $\theta$  at its centre. Radius of the arc is  $R$ .  
**Sol.** Let the arc lie in  $x$ - $y$  plane with its centre at the origin. Consider a small current element  $i d\vec{l}$  as shown in figure. The field due to this element at the centre is

$$dB = \frac{\mu_0}{4\pi} \frac{idl \sin 90^\circ}{R^2} \quad (\because id\vec{l} \text{ and } R \text{ are perpendicular})$$

Now  $d\ell = R d\phi \therefore dB = \frac{\mu_0}{4\pi} \frac{iR d\phi}{R^2} \Rightarrow dB = \frac{\mu_0}{4\pi R} i d\phi$



The direction of field is outward perpendicular to plane of paper. Total magnetic field

$$B = \int dB \therefore B = \frac{\mu_0 i}{4\pi R} \int_0^\theta d\phi$$

$$= \frac{\mu_0 i}{4\pi R} [\phi]_0^\theta \therefore B = \frac{\mu_0 i}{4\pi R} \theta$$

**Magnetic field due to a straight current carrying conductor**

Consider a straight wire carrying current  $i$ . Its ends subtend angles  $\theta_1$  and  $\theta_2$  at point  $P$  at which field is to be determined.

Take a small element  $d\vec{\ell}$  of the wire at a distance  $\ell$  from  $O$ . The magnetic field due to element is

$$dB = \frac{\mu_0}{4\pi} \frac{id\ell \sin(90^\circ + \theta)}{x^2} \dots (i)$$

The direction of field at  $P$  is perpendicular to the plane of the diagram and going into it. The direction of the field is the same for all elements of the wire and hence net field due to the wire is obtained by integrating equation (i).

From the diagram,  $\frac{\ell}{r} = \tan \theta$   
 or  $\ell = r \tan \theta$   
 and  $d\ell = r \sec^2 \theta d\theta$   
 Putting in equation (i),

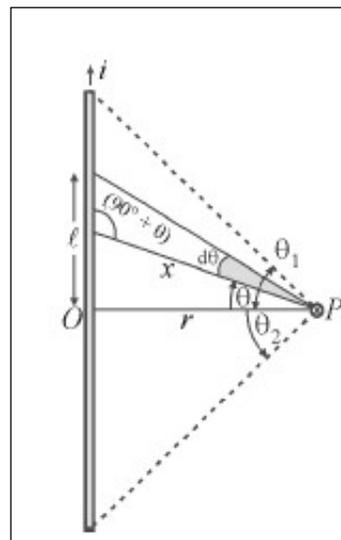
$$dB = \frac{\mu_0}{4\pi} \frac{i(r \sec^2 \theta d\theta)}{x^2} \cos \theta$$

Also  $\frac{x}{r} = \sec \theta$  or  $x = r \sec \theta$

$\therefore dB = \frac{\mu_0}{4\pi} \frac{i}{r} \cos \theta d\theta$

Total field  $B = \frac{\mu_0}{4\pi} \frac{i}{r} \int_{-\theta_2}^{\theta_1} \cos \theta d\theta$

or  $B = \frac{\mu_0}{4\pi} \frac{i}{r} (\sin \theta_1 + \sin \theta_2)$



**Special cases :**

- (a) Field due to a long straight wire (b) Field along the end of the wire

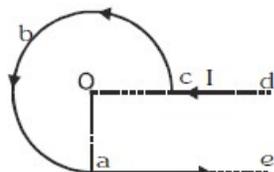
$\theta_1 = \frac{\pi}{2}$  and  $\theta_2 = \frac{\pi}{2}$

$\theta_1 = 0$  and  $\theta_2 = \pi/2$

$\therefore B = \frac{\mu_0}{2\pi} \frac{i}{r}$

$\therefore B = \frac{\mu_0}{4\pi} \frac{i}{r}$

**Example7.** Calculate the magnetic induction at the point O, if radius of the curved part of the wire is  $a$  and linear parts are assumed to be very long and parallel.



**Sol.** Magnetic induction at the point O due to circular portion of the wire is

$$B_1 = \frac{\mu_0 i \alpha}{4\pi R} = \frac{\mu_0 i}{4\pi a} \times \frac{3}{2} \pi \odot \text{ (out of the page) } (\because \alpha = \frac{3\pi}{2})$$

Magnetic induction at O due to wire cd will be zero since O lies on the line cd itself when extended backward. Magnetic induction at O due to infinitely long straight wire ae is

$$B_2 = \frac{\mu_0 i}{4\pi r} [\sin \phi_1 + \sin \phi_2] \text{ where } r = a, \phi_1 = 0, \phi_2 = \frac{\pi}{2}$$

$$\Rightarrow B_2 = \frac{\mu_0 i}{4\pi a} \left[ \sin 0^\circ + \sin \left( \frac{\pi}{2} \right) \right] = \frac{\mu_0 i}{4\pi a}$$

Because both the fields are in same direction i.e. perpendicular to plane of paper and directed upwards, hence the resultant magnetic induction at O is

$$B = B_1 + B_2 = \frac{\mu_0 i}{4\pi a} \left( \frac{3\pi}{2} + 1 \right) \odot$$

### AMPERE'S CIRCUITAL LAW

Ampere's circuital law states that the line integral of the magnetic field around any closed path in free space or vacuum is equal to  $\mu_0$  times the net current or total current which crosses through the area bounded by the closed path. Mathematically

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \Sigma I$$

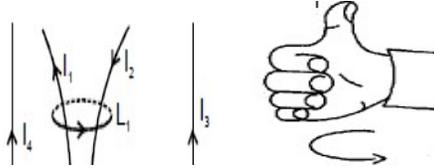
The magnetic field  $B$  on the left hand side in Ampere's law is the resultant field due to all the currents existing anywhere while on the right hand side the current  $i_{in}$  is due to the currents crossing the closed amperian loop.

Note: Ampere's circuital law holds for steady currents which do not fluctuate with time.

#### Sign convention of current for Ampere Circuital Law

If the fingers of the right-hand be curled in the sense the boundary is traversed in the loop integral.

$\oint \vec{B} \cdot d\vec{\ell}$  Then the direction of the thumb gives the sense in which the current  $I$  is regarded as positive.

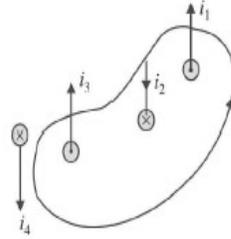


so in this case up-side directed currents are positive.

Therefore for loop L1

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 (I_1 - I_2)$$

Let us consider another loop as shown in figure.



If integration is performed along the path as shown (anticlockwise) then  $i_1$  and  $i_3$  will be positive and  $i_2$  will be negative. Thus the total current crossing the loop is  $(i_1 - i_2 + i_3)$ . Any current outside the area is not included in writing the right hand side of the equation. Thus

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 (i_1 - i_2 + i_3)$$

Note for simplicity while calculating magnetic field we should choose such an amperian loop so that

(i)  $B$  is tangential to the loop and is a non-zero constant  $B$  or (ii)  $B$  is normal to the loop, or (iii)  $B$  vanishes.

### APPLICATION OF AMPERE'S CIRCUITAL LAW

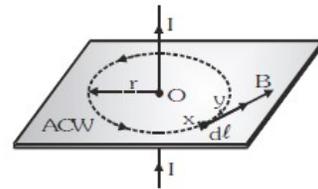
- Magnetic field due to infinite long thin current carrying straight conductor

Consider a circle of radius ' $r$ '. Let  $XY$  be the small element of length  $d\ell$ .  $\vec{B}$  and  $d\vec{\ell}$  are in the same direction because direction of  $\vec{B}$  is along the tangent of the circle. By A.C.L.

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \Sigma I, \quad \oint B d\ell \cos \theta = \mu_0 I \quad (\text{where } \theta = 0)$$

$$\oint B d\ell \cos 0^\circ = \mu_0 I \Rightarrow B \oint d\ell = \mu_0 I \quad (\text{where } \oint d\ell = 2\pi r)$$

$$B (2\pi r) = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi r}$$

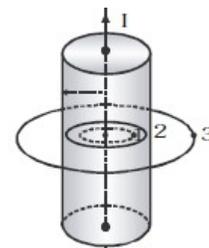


### Magnetic field due to infinite long solid cylindrical conductor

- For a point inside the cylinder  $r < R$ , Current from area  $\pi R^2$  is  $= I$

$$\text{so current from area } \pi r^2 \text{ is } = \frac{I}{\pi R^2} (\pi r^2) = \frac{I r^2}{R^2}$$

By Ampere circuital law for circular path 1 of radius  $r$



$$B_{in} (2\pi r) = \mu_0 I' = \mu_0 \frac{I r^2}{R^2} \Rightarrow B_{in} = \frac{\mu_0 I r}{2\pi R^2} \Rightarrow B_{in} \propto r$$

For a point on the axis of the cylinder ( $r = 0$ );  $B_{axis} = 0$

For a point on the surface of cylinder ( $r = R$ )

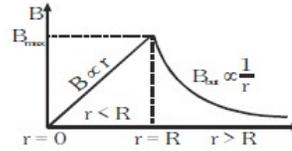
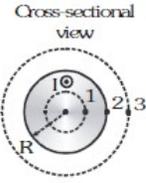
By Ampere circuital law for circular path 2 of radius R

$$B_s (2\pi R) = \mu_0 I \Rightarrow B_s = \frac{\mu_0 I}{2\pi R} \text{ (it is maximum)}$$

For a point outside the cylinder ( $r > R$ ) :-

By Ampere circuital law for circular path 3 of radius r

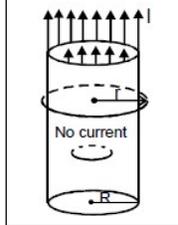
$$B_{out} (2\pi r) = \mu_0 I \Rightarrow B_{out} = \frac{\mu_0 I}{2\pi r} \Rightarrow B_{out} \propto \frac{1}{r}$$



Magnetic field outside the cylindrical conductor does not depend upon nature (thick/thin or solid/hollow) of the conductor as well as its radius of cross section.

**Magnetic field due to hollow current carrying infinitely long cylinder :**

Let us consider a hollow current carrying infinitely long cylinder of radius 'R' in which current I is uniformly distributed on the whole circumference as shown in figure.



Now we consider an amperian loop as a circle, coaxial with cylinder of radius  $r (r > R)$  as shown in figure.

By symmetry B is uniform on the loop Therefore by Ampere circuital law

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \Sigma I \quad \text{but LHS=}$$

$$\oint \vec{B} \cdot d\vec{\ell} = \oint B d\ell = B \int_0^{2\pi} d\ell = B 2\pi r,$$

Equating LHS & RHS

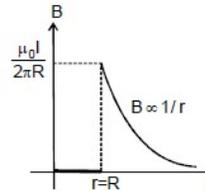
$$\text{while RHS} = \mu_0 I$$

$$B 2\pi r = \mu_0 I \text{ or } B = \mu_0 I / 2\pi r$$

But for points inside the cylinder if we draw an amperian loop as a circle, coaxial with cylinder of radius  $r (r < R)$   $I_{enc} = 0$  Therefore

$$\begin{aligned} \oint \vec{B} \cdot d\vec{\ell} &= \oint B \cdot d\ell = \mu_0 (0) \\ &= B(2\pi r) = 0 \\ \Rightarrow B_{in} &= 0 \end{aligned}$$

The graph below illustrates the variation of magnetic field due to a hollow current carrying infinitely long cylinder of radius 'R' with distance r from its axis.



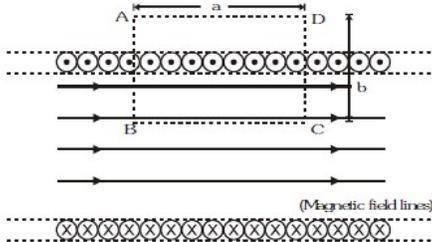
**Magnetic field due to a long solenoid**

A solenoid is a tightly wound helical coil of wire.

Figure shows a part of long solenoid with number of turns/length = n.

We can find the magnetic field by using Ampere circuital law.

The figure below is vertical cross section of the above solenoid in which we consider a rectangular loop ABCD.



For this loop by Ampere circuital law

$$\oint_{ABCD} \vec{B} \cdot d\vec{\ell} = \mu_0 i_{enc}$$

$$\text{Now } \oint_{ABCD} \vec{B} \cdot d\vec{\ell} = \oint_{AB} \vec{B} \cdot d\vec{\ell} + \oint_{BC} \vec{B} \cdot d\vec{\ell} + \oint_{CD} \vec{B} \cdot d\vec{\ell} + \oint_{DA} \vec{B} \cdot d\vec{\ell} = B \times a$$

$$\text{This is because } \oint_{AB} \vec{B} \cdot d\vec{\ell} = \oint_{CD} \vec{B} \cdot d\vec{\ell} = 0, \vec{B} \perp d\vec{\ell}.$$

$$\text{And, } \oint_{DA} \vec{B} \cdot d\vec{\ell} = 0 \text{ ( } \because \vec{B} \text{ outside the solenoid is negligible)}$$

$$\text{Now, } i_{enc} = (n \times a) \times i$$

$$B \times a = \mu_0 (n \times a \times i) \Rightarrow B = \mu_0 n i$$

**Example 8.** A closely wound, solenoid 80 cm. long has 5 layers of winding of 400 turns each. The diameter of the solenoid is 1.8 cm. If the current carried is 8.0A. Estimate the magnetic field inside the solenoid.

**Soln.** Magnetic field within solenoid is  $B_{in} = \mu_0 n I = \mu_0 N I / L$   
 But  $N = 400 \times 5 = 2000$ . Hence  $B_{in} = \frac{4\pi \times 10^{-7} \times 2000 \times 8}{(80 \times 10^{-2})} = 8\pi \times 10^{-3} \text{ T}$

### The toroid

The toroid is a hollow circular ring on which a large number of turns of a wire are closely wound. It can be viewed as a solenoid which has been bent into a circular itself. It is shown in Fig. carrying a current  $I$ .

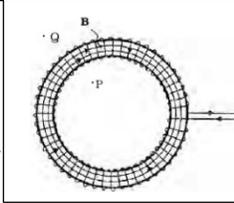
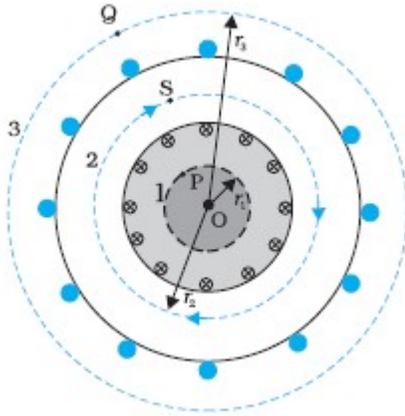


Figure below shows a sectional view of the toroid.



The direction of the magnetic field inside is clockwise as per the right-hand thumb rule for circular loops. Three circular Amperian loops 1, 2 and 3 are shown by dashed lines. By symmetry, the magnetic field should be tangential to each of them and constant in magnitude for a given loop. The circular areas bounded by loops 2 and 3 both cut the toroid: so that each turn of current carrying wire is cut once by the loop 2 and twice by the loop 3.

Let the magnetic field along loop 1 be  $B_1$  in magnitude. Then by Ampere's circuital law since the loop encloses no current, so  $B_1 (2\pi r_1) = \mu_0(0)$ , Therefore  $B_1 = 0$ . Thus, the magnetic field at any point P in the open space inside the toroid is zero.

We shall now show that magnetic field at Q is likewise is zero. Let the magnetic field along loop 3 be  $B_3$ . Through loop 3 the current coming out of the plane of the paper is cancelled exactly by the current going into it. Once again from Ampere's law

$$B_3 (2\pi r_3) = \mu_0(0), \text{ Therefore } B_3 = 0$$

Let the magnetic field inside the solenoid be  $B$ . We shall now consider the magnetic field at S. Once again we employ Ampere's law in the loop 2 of radius  $r$ . The current enclosed  $I_{enc}$  is  $NI$  (for  $N$  turns of toroidal coil).

$$\text{By Ampere's law } B (2\pi r) = \mu_0 NI$$

$$\text{Therefore } B = \frac{\mu_0 NI}{2\pi r}$$

Let  $r$  be the average radius of the toroid and  $n$  be the number of turns per unit length. Then

$$N = 2\pi r n = (\text{average perimeter of the toroid}) \times \text{number of turns per unit length}$$

$$\text{and thus, } B = \mu_0 n I$$

### Magnetic force on current element

When current element  $I d\vec{l}$  is placed in magnetic field  $\vec{B}$  then, it experiences a magnetic force

$$d\vec{F}_m = I(d\vec{l} \times \vec{B})$$

Total magnetic force on straight current carrying conductor in uniform magnetic field given as

$$\vec{F}_m = I(\vec{L} \times \vec{B})$$

Its magnitude is  $F = ILB \sin\theta$  where  $\theta$  is angle between current element and magnetic field.

Its direction is given by Fleming's Left Hand rule which states that if we stretch thumb, index and middle finger of left hand mutually perpendicular such that, index finger points magnetic field, middle finger towards component of current element perpendicular to magnetic field, then force on conductor is given by direction of thumb.

- Current element in a magnetic field does not experience any force if the current in it is parallel or anti-parallel with the field, that is when  $\theta = 0^\circ$  or  $180^\circ$
- Current element in a magnetic field experiences maximum force if the current in it is perpendicular with the field, that is if  $\theta = 90^\circ$
- Magnetic force on current element is always perpendicular to the current element vector and magnetic field vector.

### FORCE BETWEEN TWO PARALLEL CURRENTS, THE AMPERE

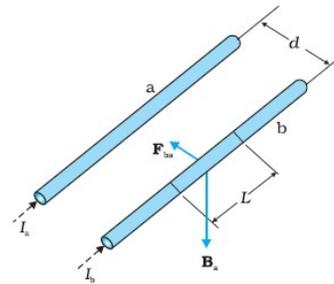


Figure shows two long parallel conductors a and b separated by a distance  $d$ , placed horizontally and carrying (parallel) currents  $I_a$  and  $I_b$ , respectively. The conductor 'a' produces, the same magnetic field  $B_a$  at all points along the conductor 'b'. The right-hand rule tells us that the direction of this field is downwards. Its magnitude

$$\text{is given by } B_a = \frac{\mu_0 I_a}{2\pi d}$$

The conductor 'b' carrying a current  $I_b$  will experience a sideways force due to the field  $B_a$ . The direction of this force is towards the conductor 'a'. We label this force as  $F_{ba}$ , the force on a segment  $L$  of 'b' due to 'a'. The magnitude of this force is given by

$$F_{ba} = I_b L B_a = \frac{\mu_0 I_a I_b L}{2\pi d}$$

Similarly the force on 'a' due to 'b', on a segment of length  $L$  of 'a' due to the current in 'b' is equal in magnitude to  $F_{ba}$ , and directed towards 'b'. Thus,

$$\vec{F}_{ba} = -\vec{F}_{ab}$$

Note that this is consistent with Newton's third Law.

We have seen from above that currents flowing in the same direction attract each other.

Thus, *Parallel currents attract, and antiparallel currents repel.*

Let  $f_{ba}$  represent the magnitude of the force  $F_{ba}$  per unit length. Then  $f_{ba} = \frac{\mu_0 I_a I_b}{2\pi d}$

The above expression is used to define the ampere (A). The *ampere* is the value of that steady current which, when maintained in each of the two very long, straight, parallel conductors of negligible cross-section, and placed one metre apart in vacuum, would produce on each of these conductors a force equal to  $2 \times 10^{-7}$  newtons per metre of length.

**Example 9:** The magnetic field at a certain place is  $6.0 \times 10^{-5}$  T directed horizontally from south to north. A very long straight conductor is carrying a steady current of 0.5A. What is the magnetic force per unit length on it when it is placed on a table having current from east to west?

Solution  $F = I l B \sin\theta$

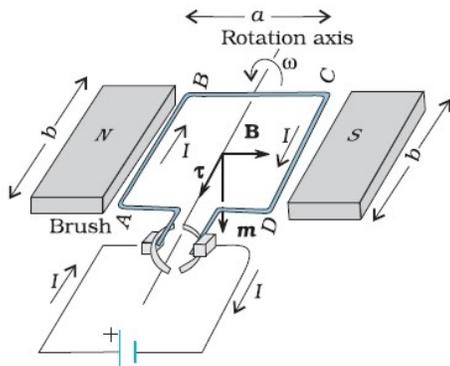
The force per unit length is  $f = F/l = I B \sin\theta$

(a) When the current is flowing from east to west,  $\theta = 90^\circ$

Hence,  $f = I B = (1/2) \times 6 \times 10^{-5} = 3 \times 10^{-5} \text{ N m}^{-1}$

The direction of the force is downwards by Fleming's Left Hand Rule.

### Torque on a rectangular current loop in a uniform magnetic field



We consider rectangular loop carrying a steady current  $I$  placed such that the uniform magnetic field  $B$  is in the plane of the loop as shown in Fig.

The field exerts no force on the two arms AD and BC of the loop since current is flowing parallel to magnetic field. Magnetic force  $F_1$  is perpendicular to the arm AB of the loop which is directed into the plane of the loop. Its magnitude is,  $F_1 = I b B$

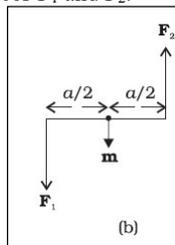
Similarly magnetic force  $F_2$  on the arm CD is directed out of the plane of the paper.  $F_2 = I b B = F_1$

Thus, the *net force* on the loop is zero. There is a torque on the loop due to the pair of forces  $F_1$  and  $F_2$ .

Figure(b) shows a view of the loop from the AD end. It shows that the torque on the loop tends to rotate it anti-clockwise. This torque is (in magnitude),

$$\tau = F_1 \frac{a}{2} + F_2 \frac{a}{2}$$

$$= I b B \frac{a}{2} + I b B \frac{a}{2} = I (ab) B$$



$= I A B$  where  $A = ab$  is the area of the rectangle.

When the plane of the loop, is not along the magnetic field, but makes an angle  $\theta$  with it.

The magnitude of the torque on the loop is,

$$\tau = F_1 \frac{a}{2} \sin\theta + F_2 \frac{a}{2} \sin\theta$$

$$= I a b B \sin\theta$$

$$= I A B \sin\theta$$

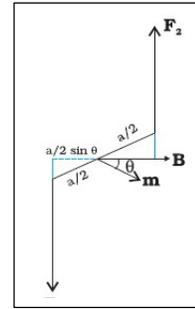
We define the *magnetic moment* of the

current loop as,  $\mathbf{m} = I \mathbf{A}$  where the direction of the area vector  $\mathbf{A}$  or *magnetic moment* is given by the right-hand thumb rule which states that if we curl four fingers of our right hand as per the flow of current in the loop, then the thumb which was held perpendicular to other fingers give the direction of magnetic moment.

Thus magnitude of torque in above Equation can be expressed as  $\tau = m B \sin\theta$  since  $m = I A$

Or in vectors torque can be expressed as vector product of the magnetic moment of the coil and the magnetic field as

$$\boldsymbol{\tau} = \mathbf{m} \times \mathbf{B}$$



Note

- 1) The dimensions of the magnetic moment are  $[AL^2]$  and its unit is  $\text{Am}^2$ .
- 2) When  $\mathbf{m}$  and  $\mathbf{B}$  are parallel the equilibrium is a stable one.
- 3) When  $\mathbf{m}$  and  $\mathbf{B}$  are antiparallel, the equilibrium is unstable
- 4) A small magnet or any magnetic dipole which is free to rotate aligns itself with the external magnetic field.
- 5) If the loop has  $N$  closely wound turns, the expression for magnetic moment is  $\mathbf{m} = N I \mathbf{A}$

### Circular current loop as a magnetic dipole

The magnetic field (at large distances) due to current in a circular current loop is very similar in behavior to the electric field of an electric dipole. This similarity shows that current loop can be considered as magnetic dipole. We know that the magnetic field on the axis of a circular loop, of a radius  $R$ , carrying a steady current  $I$  in magnitude is given by

$$B = \frac{\mu_0 I R^2}{2(x^2 + R^2)^{3/2}}$$

and its direction is along the axis and given by the right-hand thumb rule.

For  $x \gg R$ , we may drop the  $R^2$  term in the denominator.

Also since the area of the loop  $A = \pi R^2$ .  $\therefore R^2 = A/\pi$  Thus

$$B = \frac{\mu_0 I A}{2\pi x^3}$$

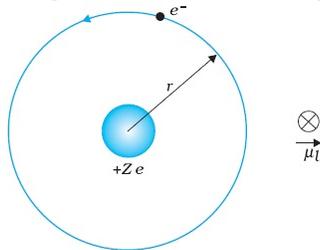
But the magnetic moment  $\mathbf{m}$  have a magnitude  $IA$ , Therefore

$$\mathbf{B} = \frac{\mu_0 \mathbf{m}}{2\pi x^3} = \frac{\mu_0}{4\pi} \frac{2\mathbf{m}}{x^3}$$

which is a result similar to The magnetic dipole moment labelled as  $\boldsymbol{\mu}_l$  is known as *orbital magnetic moment*. Besides the orbital moment, the electron has an *intrinsic* magnetic moment, called the

that obtained for electric field due to dipole on its axis. Hence we can say a current carrying loop behaves as a magnetic dipole.

### The magnetic dipole moment of a revolving electron



Let us consider an electron of charge  $(-e)$  performing uniform circular motion in a circular path of radius ' $r$ ', with speed ' $v$ ' and time period ' $T$ '. This constitutes a current  $I$ , where,

$$I = \frac{e}{T} \quad \text{and} \quad T = \frac{2\pi r}{v}$$

$\therefore$  we have  $I = ev/2\pi r$ .

There will be a magnetic moment, usually denoted by  $\mu_l$ , associated with this circulating current. Whose magnitude is,  $\mu_l = I\pi r^2 = evr/2$ .

The direction of this magnetic moment is into the plane of the paper in above Fig.

Multiplying and dividing the right-hand side of the above expression by the electron mass  $m_e$ , we have,

$$\mu_l = \frac{e}{2m_e} (m_e v r) = \frac{e}{2m_e} l$$

Here,  $l$  is the magnitude of the angular momentum of the electron about the central nucleus ("orbital" angular momentum). Vectorially,

$$\mu_l = -\frac{e}{2m_e} \mathbf{l}$$

The negative sign indicates that the angular momentum of the electron is opposite in direction to the magnetic moment.

Instead of electron with charge  $(-e)$ , if we had taken a particle with charge  $(+q)$ , the angular momentum and magnetic moment would be in the same direction.

The ratio

$\frac{\mu_l}{l} = \frac{e}{2m_e}$  is called the **gyromagnetic ratio** and is a constant. Its value is  $8.8 \times 10^{10}$  C/kg for an electron.

Bohr hypothesised that the angular momentum assumes a discrete set of values, namely,

$$l = \frac{nh}{2\pi} \quad \text{where } n \text{ is a natural number, } n = 1, 2, 3, \dots$$

and  $h$  is a constant named after Max Planck (Planck's constant) with a value  $h = 6.626 \times 10^{-34}$  J s.

This condition of discreteness is called the **Bohr quantisation condition**. Take the value  $n = 1$ , we have

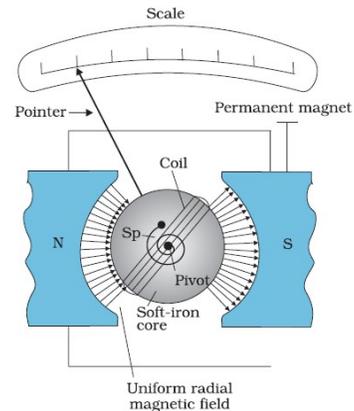
$$(\mu_l)_{\min} = \frac{e}{4\pi m_e} h = \frac{1.60 \times 10^{-19} \times 6.63 \times 10^{-34}}{4 \times 3.14 \times 9.11 \times 10^{-31}} = 9.27 \times 10^{-24} \text{ Am}^2$$

This value is called the

**Bohr magneton**.

*spin magnetic moment.*

### THE MOVING COIL GALVANOMETER (MCG)



It consists of a coil, with many turns, free to rotate about a fixed axis, in a uniform radial magnetic field. There is a cylindrical soft iron core which not only makes the field radial but also increases the strength of the magnetic field. When a current flows through the coil, a torque acts on it. This torque is given by  $\tau = NIAB$

where the symbols have their usual meaning. Since the field is radial by design, we have taken  $\sin\theta = 1$  in the above expression for the torque.

The magnetic torque  $NIAB$  tends to rotate the coil. A spring  $S_p$  provides a counter torque  $k\phi$  that balances the magnetic torque  $NIAB$ ; resulting in a steady angular deflection  $\phi$ .

In equilibrium  $k\phi = NIAB$

where  $k$  is the torsional constant of the spring; i.e. the restoring torque per unit twist. The deflection  $\phi$  is indicated on the scale by a pointer attached to the spring. We have

$$\phi = \left( \frac{NAB}{k} \right) I$$

The quantity in brackets is galvanometer constant.

We define the **current sensitivity** of the galvanometer as the deflection per unit current

$\frac{\phi}{I} = \frac{NAB}{k}$  A convenient way for the manufacturer to increase the sensitivity is to increase the number of turns  $N$ .

We define the **voltage sensitivity** as the deflection per unit voltage.

$$\frac{\phi}{V} = \left( \frac{NAB}{k} \right) \frac{I}{V} = \left( \frac{NAB}{k} \right) \frac{1}{R}$$

An interesting point to note is that increasing the current sensitivity by increasing  $N$  may not necessarily increase the voltage sensitivity, which would remain unchanged as the resistance of the galvanometer is proportional to the length of the wire and hence  $N$ .

## SHUNT

The small resistance connected in parallel to galvanometer coil, in order to control current flowing through the galvanometer, is known as shunt.

### CONVERSION OF GALVANOMETER INTO AMMETER

A galvanometer can be converted into an ammeter by connecting low resistance in parallel to its coil.

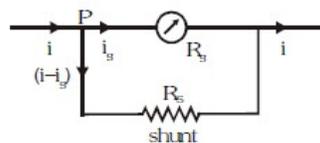
- The value of shunt resistance to be connected in parallel to galvanometer

$$\text{coil is given by : } R_s = \frac{R_g i_g}{i - i_g}$$

Where  $i$  = Range of ammeter

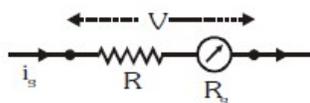
$i_g$  = Current required for full scale deflection of galvanometer.

$R_g$  = Resistance of galvanometer coil.



### CONVERSION OF GALVANOMETER INTO VOLTMETER

- The galvanometer can be converted into voltmeter by connecting high resistance in series with its coil.
- The high resistance to be connected in series with galvanometer coil is given by  $R = \frac{V}{i_g} - R_g$



## HOME ASSIGNMENT (4)

- A length  $L$  of wire carries a steady current  $I$ . It is bent first to form a circular plane coil of one turn. The same length is now bent more sharply to give a double loop of smaller radius. The magnetic field at the centre caused by the same current is
  - A quarter of its first value
  - Unaltered
  - Four times of its first value
  - A half of its first value
- A vertical straight conductor carries a current vertically upwards. A point  $P$  lies to the east of it at a small distance and another point  $Q$  lies to the west at the same distance. The magnetic field at  $P$  is [neglecting earth magnetism]
  - Greater than at  $Q$
  - Same as at  $Q$
  - Less than at  $Q$
  - Greater or less than at  $Q$  depending upon the strength of the current
- If a copper rod carries a direct current, the magnetic field associated with the current will be
  - Only inside the rod
  - Only outside the rod
  - Both inside and outside the rod
  - Neither inside nor outside the rod
- If a long hollow copper pipe carries a direct current, the magnetic field associated with the current will be
  - Only inside the pipe
  - Only outside the pipe
  - Neither inside nor outside the pipe
  - Both inside and outside the pipe
- Field at the centre of a circular coil of radius  $r$ , through which a current  $I$  flows is
  - Directly proportional to  $r$
  - Inversely proportional to  $I$
  - Directly proportional to  $I$
  - Directly proportional to  $I^2$
- The magnetic field  $B$  within long solenoid having  $n$  turns per metre length and carrying a current of  $i$  ampere is given by
  - $\frac{\mu_0 n i}{e}$
  - $\mu_0 n i$
  - $4\pi\mu_0 n i$
  - $n i$
- Field inside a solenoid is [MP PMT 1993]
  - Directly proportional to its length
  - Directly proportional to current
  - Inversely proportional to total number of turns
  - Inversely proportional to current
- A particle carrying a charge equal to 100 times the charge on an electron is rotating per second in a circular path of radius 0.8 metre. The value of the magnetic field produced at the centre will be
  - $\frac{10^{-7}}{\mu_0}$
  - $10^{-17} \mu_0$
  - $10^{-6} \mu_0$
  - $10^{-7} \mu_0$  □

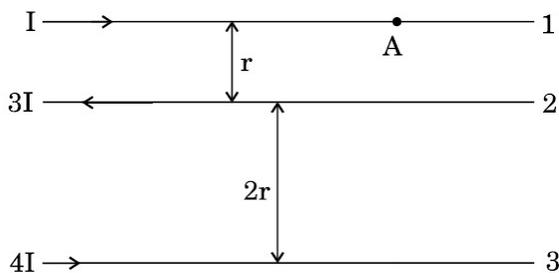
<p>9. A vertical wire carries a current in upward direction. An electron beam sent horizontally towards the wire will be deflected</p> <p>(a) towards right (b) towards left (c) upwards (d) downwards.</p> <p>10. A current-carrying, straight wire is kept along the axis of a circular loop carrying a current. The straight wire</p> <p>(a) will exert an inward force on the circular loop (b) will exert an outward force on the circular loop (c) will not exert any force on the circular loop (d) will exert a force on the circular loop parallel to itself.</p> <p>11. A proton beam is going from north to south and an electron beam is going from south to north. Neglecting the earth's magnetic field, the electron beam will be deflected</p> <p>(a) towards the proton beam (b) away from the proton beam (c) upwards (d) downwards.</p> <p>12. A circular loop is kept in that vertical plane which contains the north-south direction. It carries a current that is towards north at the topmost point. Let <math>A</math> be a point on the axis of the circle to the east of it and <math>B</math> a point on this axis to the west of it. The magnetic field due to the loop</p> <p>(a) is towards east at <math>A</math> and towards west at <math>B</math> (b) is towards west at <math>A</math> and towards east at <math>B</math> (c) is towards east at both <math>A</math> and <math>B</math> (d) is towards west at both <math>A</math> and <math>B</math>.</p> <p>13. A charged particle is moved along a magnetic field line. The magnetic force on the particle is</p> <p>(a) along its velocity (b) opposite to its velocity (c) perpendicular to its velocity (d) zero.</p> <p>14. A moving charge produces</p> <p>(a) electric field only (b) magnetic field only (c) both of them (d) none of them</p> <p>15. Two parallel wires carry currents of 20 A and 40 A in opposite directions. Another wire carrying a current antiparallel to 20 A is placed midway between the two wires. The magnetic force on it will be</p> <p>(a) towards 20 A (b) towards 40 A (c) zero (d) perpendicular to plane of the currents.</p> <p>16. Two charged particles traverse identical helical paths in a completely opposite sense in a uniform magnetic field <math>\mathbf{B} = B_0 \mathbf{k}</math></p> <p>(a) They have equal z-components of momenta. (b) They must have equal charges. (c) They necessarily represent a particle-antiparticle pair. (d) The charge to mass ratio satisfy :</p> $\left(\frac{e}{m}\right)_1 + \left(\frac{e}{m}\right)_2 = 0$	<p>17. Biot-Savart law indicates that the moving electrons (velocity <math>\mathbf{v}</math>) produce a magnetic field <math>\mathbf{B}</math> such that</p> <p>(a) <math>\mathbf{B} \perp \mathbf{v}</math>. (b) <math>\mathbf{B} \parallel \mathbf{v}</math>. (c) it obeys inverse cube law. (d) it is along the line joining the electron and point of observation</p> <p>18. A current carrying circular loop of radius <math>R</math> is placed in the <math>x</math>-<math>y</math> plane with centre at the origin. Half of the loop with <math>x &gt; 0</math> is now bent so that it now lies in the <math>y</math>-<math>z</math> plane.</p> <p>(a) The magnitude of magnetic moment now diminishes. (b) The magnetic moment does not change. (c) The magnitude of <math>B</math> at <math>(0,0,z)</math>, <math>z \gg R</math> increases. (d) The magnitude of <math>B</math> at <math>(0,0,z)</math>, <math>z \gg R</math> is unchanged.</p> <p>19. An electron is projected with uniform velocity along the axis of a current carrying long solenoid. Which of the following is true?</p> <p>(a) The electron will be accelerated along the axis. (b) The electron path will be circular about the axis. (c) The electron will experience a force at <math>45^\circ</math> to the axis and hence execute a helical path. (d) The electron will continue to move with uniform velocity along the axis of the solenoid.</p> <p>20. In a cyclotron, a charged particle</p> <p>(a) undergoes acceleration all the time. (b) speeds up between the dees because of the magnetic field. (c) speeds up in a dee. (d) slows down within a dee and speeds up between dees.</p> <p>21. A circular current loop of magnetic moment <math>M</math> is in an arbitrary orientation in an external magnetic field <math>B</math>. The work done to rotate the loop by <math>30^\circ</math> about an axis perpendicular to its plane is</p> <p>(a) <math>MB</math>. (b) <math>\frac{\sqrt{3}}{2} MB</math>. (c) <math>\frac{MB}{2}</math>. (d) zero.</p> <p>22. An <math>\alpha</math>-particle and a proton of the same kinetic energy are in turn allowed to pass through a magnetic field <math>B</math>, acting normal to the direction of motion of the particles. Calculate the ratio of radii of the circular paths described by them.</p> <p>23. A proton and an electron travelling along parallel paths enter a region of uniform magnetic field, acting perpendicular to their paths. Which of them will move in a circular path with higher frequency ?</p> <p>24. Two protons of equal kinetic energies enter a region of uniform magnetic field. The first proton enters normal to the field direction while the second enters at <math>30^\circ</math> to the field direction. Name the trajectories followed by them.</p> <p>25. State Biot – Savart law and express it in the vector</p>
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form. Using Biot – Savart law, obtain the expression for the magnetic field due to a circular coil of radius  $r$ , carrying a current  $I$  at a point on its axis distant  $x$  from the centre.

26. Two long straight parallel conductors carrying steady currents  $I_a$  and  $I_b$  along the same direction are separated by a distance  $d$ . How does one explain the force of attraction between them? If a third conductor carrying a current  $I_c$  in the opposite direction is placed just in the middle of these conductors, find the resultant force acting on the third conductor.

27. Define SI unit of current in terms of the force between two parallel current carrying conductors.

28. The figure shows three infinitely long straight parallel current carrying conductors. Find the  
(i) magnitude and direction of the net magnetic field at point A lying on conductor 1,  
(ii) magnetic force on conductor 2.



29. State the Lorentz's force and express it in vector form. Which pair of vectors are always perpendicular to each other? Derive the expression for the force acting on a current carrying conductor of length  $L$  in a uniform magnetic field 'B'.

30. A charge  $q$  of mass  $m$  is moving with a velocity of  $V$ , at right angles to a uniform magnetic field  $B$ . Deduce the expression for the radius of the circular path it describes.

31. Define the term current sensitivity of a galvanometer.

32. State the condition under which a charged particle moving with velocity  $v$  goes undeflected in magnetic field  $B$ .

33. An electron, after being accelerated through a potential difference of  $104$  V, enters a uniform magnetic field of  $0.04$  T, perpendicular to its direction of motion. Calculate the radius of curvature of its trajectory.

34. (i) With the help of a neat and labelled diagram, explain the principle and working of a moving coil galvanometer.

(ii) What is the function of uniform radial field and how is it produced?

(iii) How is current sensitivity increased?

35. A neutron, an electron and an alpha particle moving with equal velocities, enter a uniform magnetic field going into the plane of the paper as shown. Trace their

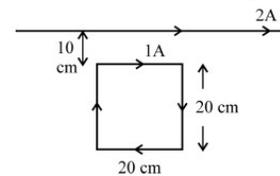
paths in the field and justify your answer.



36. State Ampere's circuital law. Use this law to obtain the expression for the magnetic field inside an air cored toroid of average radius ' $r$ ', having ' $n$ ' turns per unit length and carrying a steady current  $I$ .

37. An observer to the left of a solenoid of  $N$  turns each of cross section area ' $A$ ' observes that a steady current  $I$  in it flows in the clockwise direction. Depict the magnetic field lines due to the solenoid specifying its polarity and show that it acts as a bar magnet of magnetic moment  $m = NIA$ .

38. A square loop of side  $20$  cm carrying current of  $1A$  is kept near an infinite long straight wire carrying a current of  $2A$  in the same plane as shown in the figure.



Calculate the magnitude and direction of the net force exerted on the loop due to the current carrying conductor.

39. A square shaped plane coil of area  $100$  cm<sup>2</sup> of  $200$  turns carries a steady current of  $5A$ . It is placed in a uniform magnetic field of  $0.2$  T acting perpendicular to the plane of the coil. Calculate the torque on the coil when its plane makes an angle of  $60^\circ$  with the direction of the field. In which orientation will the coil be in stable equilibrium?

40. Derive an expression for the magnetic moment of an electron revolving around the nucleus in terms of its angular momentum.

What is the direction of the magnetic moment of the electron with respect to its angular momentum?